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A Hilbert-Space Framework for the Genesis of Conscious Mental States

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Abstract

Starting from Jung's psychoanalytical perspective on mind and building on recently developed cognitive systems theory based on the mathematics of a Hilbert space, a formalism is developed in which mental states – conscious as well as unconscious – are described by state vectors in a Hilbert space. Particularly, it is proposed that the mental process of metabolization of blurred, semi-conscious, or dreamlike mental states into clear ideas and unambiguous cognitive states can be formalized using eigenvalue equations. A comparison between experiences from psychoanalytical clinical practice and the phenomena to be expected from the proposed model are presented. Also, correspondences between the model and results from neuroscientific research on brain-hemisphere functionality are identified. Implications for clinical methodology are suggested.

1. Introduction

In 1938 the Swiss psychiatrist Carl Gustav Jung, who had an intense collaboration with Wolfgang Pauli, wrote to another colleague:[1]

> I would not be surprised if one day we saw a far reaching agreement between psychology and physics. I am convinced that if the two sciences pursue their goals with utmost consistency and right into the ultimate depth of man they must hit upon a common formula.

Recent publications in cognitive systems theory in which the mental apparatus is formalized by a type of mathematics which is similar to the Hilbert Space mathematics on which quantum physics is based are indeed pointing in this direction.[2] Recently, Blutner and Hochnadel (2010) presented a four-dimensional Hilbert-space model[3] for Jung's sixteen differ-

[1] Jung in a letter to Oscar Hug, May 24, 1938 (Adler and Jaffé 1953, p. 246).

[2] For instance, compare the work of Aerts *et al.* (2006), Atmanspacher *et al.* (2002), Blutner (2009), Busemeyer *et al.* (2006), Franco (2007), Gabora *et al.* (2008), Khrennikov (2003), Pothos and Busemeyer (2009).

[3] Blutner and Hochnadel treat the two rational functions of Jung's personality theory (thinking T and feeling F) as compatible operators $T = \sigma_z, F = -\sigma_z$ (and the same for Jung's two irrational functions sensation S and intuition I; their formula 21). In fact, only operators corresponding to observables that commute are compatible. This would require that T and F (and S and I) would possess a complete orthonormal set of common eigenfunctions which they clearly do not. This does not invalidate their approach in general, just their assignment.

ent personality types which shows why the present practice of personality diagnostics cannot be based on classical statistics alone.

The present paper proposes a further step in the formalization of mental phenomena by Hilbert-space mathematics. It essentially takes a psychoanalytic perspective which includes the consideration of unconscious mental states. Particularly, a formal description is proposed for the transformation of unconscious states with blurred, ungrasped content into conscious mental states with clearly grasped content.

Mental phenomena can roughly be pigeonholed in two categories. The first one consists of blurred, ambiguous, dream-like states whose contents have not been consciously grasped and in which space, time and logic are not clearly defined. The second one consists of conscious experiences consisting of clear ideas and images. This is the realm of unambiguity, understanding and knowledge in which a clear sense of time and space exists and where logic applies. A successful psychodynamic psychotherapy is accompanied by the transformation of psychic material from the first to the second category. This transformation, or metabolization[4], of ungrasped contents into grasped ones is clinically relevant.

Therefore, in psychoanalytical literature the world of the unconscious and the interaction between ungrasped and grasped phenomena have been the subject of much research. Freud spoke of the unconscious as the realm of the unlogical (Freud 1938, p. 168). Jung's entire oeuvre is dedicated to said transformation (see, e.g., Jung 1956). Jung speaks of "the spirit of this time" and "the spirit of the depth" (Jung 2009), the former of which he calls sense ("Sinn"), the latter nonsense ("Widersinn").

Bion (1962) introduced the idea of β -elements (proto-mental phenomena) and α-elements (thoughts that can be thought by the thinker) and studied the transformational dynamics between them. Matte-Blanco (1988) proposed that, contrary to the familiar logic of reason, the deeper strata of psyche follow a different logic which he called "bilogic". In cognitive psychology, the mental process of conceptualization involves comparison of the mental content to be conceptualized – *exemplars* – in terms of its *similarity* to *prototypes* (Margolis and Laurence 1999).

The present work is the result of an exploration which started from the question whether Hilbert-space mathematics, the parallels of which with mental phenomena are becoming increasingly clear, can be used to further the understanding of the psychoanalytically important transformation from unmetabolized to metabolized mental states.

[4]Throughout this paper we will use the term *metabolization* which in its psychoanalytical sense is not necessarily a familiar notion in cognitive psychology. The term was coined by Bion (1962) to describe the transformation from proto-mental psychic material to "thoughts that can be thought". *Metabolization* seems to describe exactly the experience that patients have when coming to insights. *Metabolization* relates not only to *cognition* but also to *re-cognition* (particularly of emotionally charged states).

This paper starts with some fundamental issues in the interpretation of present-day quantum theory, which involve a transformation very similar to said transformation between mental states. Thereafter, the idea of a "fabric of psyche" will be introduced. Some very basic properties of Hilbert spaces will be presented and a proposal for modeling the phenomenology of the psyche, particularly the process of mental metabolization, will be presented in terms of the mathematics of Hilbert space. The paper concludes with some ideas about the possible experimental verification and about the implications of the proposed model for clinical practice.

2. Some Fundamental Issues in Present-Day Quantum Mechanics

In quantum physics the idea of an objective world in which objects possess unambiguous properties independent of their observation has become untenable. Contrary to classical physics, results from experiments have forced quantum physicists not to speak of objects *as they are* but only about objects *as they appear*. Quantum theory even predicted (see, e.g., Bell 1987) that the realist ontology in its strictest form of the reality of a material world is flawed (see, e.g., d'Espagnat 2006, pp. 58 ff). The assumption that a subatomic system, say a bundle of electrons, has a clearly defined physical state irrespective of its observation is in contradiction to results of recent experiments (see Aspect *et al.* 1982).

Quantum systems are described mathematically by so-called state vectors (or wavefunctions) obeying the rules of Hilbert space. According to the most widely accepted interpretation, quantum mechanics describes our knowledge of physical systems and leaves the question about the existence of the state of the system as an objective outer reality unanswered. In quantum mechanics (our knowledge of) physical systems just before being observed is described by state vectors which can be written mathematically as a summation – also called superposition – of primary vectors. Physical systems (or our understanding of them) just after being observed are described by state vectors that are no longer superpositions but have been reduced to a state which corresponds to one of the primary vectors. The state vector which was in a state of superposition, i.e. a state of different possible outcomes, before observation is said to have *collapsed* into a single state as the observer performs her measurement. This is schematically illustrated in Fig. 1.

Somewhere on the way between the physical system and the registration of the measurement result, the state vector collapses. The question as to where the transformation from a superposition to an unambiguous collapsed state takes place is not prescribed or specified by the rules of

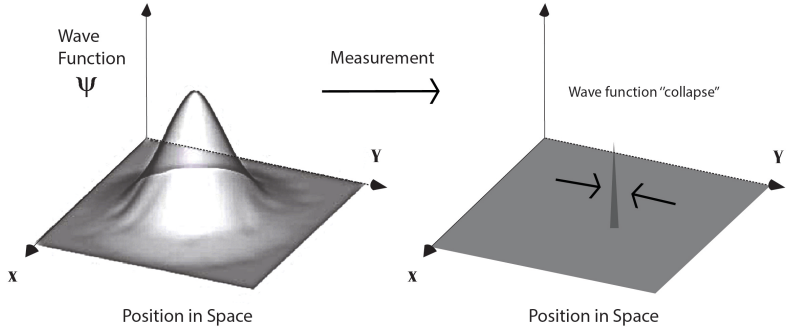

Figure 1: An example of the collapse of the state vector (or wavefunction).

quantum physics. Physicists loosely refer to this question by a "cut" between the observed system and its environment. A variety of theories, situating this cut in the physical system under observation or in the neural network of an observer or somewhere between brain and mind have been put forward. None of these theories, however, has given a decisive answer. Most, if not all of them, have been developed from the physical perspective. Psychological approaches, and particularly approaches from the viewpoint of analytical psychology and the idea of an objective psyche, are very rare indeed.

3. The Carrier-Wave Model of Psyche and the Reality of Psyche

In this paper mental manifestations are considered as phenomena emerging from the mental apparatus which in turn is considered to have at least in part objective properties. We are using the perspective of Jung, which was essentially a phenomenological one (Brooke 1991).[5] Jung wrote about the deeper strata of the psyche as a collective and objective inner reality which he considered at least as real as the material outside reality (Jung 1933, par. 683):

> We could well point to the idea of psychic reality as the most important achievement of modern psychology if it were recognized as

[5] Jung's work was not necessarily phenomonelogical in the strict philosophical sense. In philosophy, the discipline of phenomenology aims at a scientific exploration of subjective experience (see, e.g., Giorgi 1970). Phenomenology originates from the work of Edmund Husserl (1858–1938) and explores subjective experience as phenomena in their own right. It emphasizes the need to distinguish between the uncolored observation of the psychic phenomenon itself and its interpretation. An important element of the phenomenological approach is the so called "eidetic reduction". See Husserl (1913).

> such. It seems to me only a question of time for this idea to be generally accepted.

Here Jung emphasizes the difficulty of recognizing psyche as a phenomenon in its own right. Possibly because of this difficulty, physicists exploring the frontiers of the interpretation of quantum physics have, as far as we know, never considered the psyche as a reality and have tended to restrict the observer in the laboratory to her material aspects (such as the brain and the neural system).

From a phenomenological perspective the question is justified if mental phenomena exhibit regularities and patterns that can be mathematically modeled in the same way physics models the material world. On the other hand, the perspective of the psychoanalyst suggests the question how the psyche with its poetic and mythological strata and complex inter-human dynamics can ever be expressed in rational mathematical formulas. The proposal made here is that the psyche can be modeled mathematically while not losing its illogical poetry. The *fabric of psyche* can be imagined as a fundamental stratum carrying psychological content on top of it in the same way that in radio technology a carrier-wave carries the sound-wave of what is broadcasted. For example, when a piece of music is broadcasted, the radio-station transmits a carrier-wave on top of which a sound-wave of this music is modulated. This is illustrated in Fig. 2.

In this analogy, the fabric of psyche relates to the contents of psyche as the carrier-wave relates to the sound-wave. The fabric of psyche is

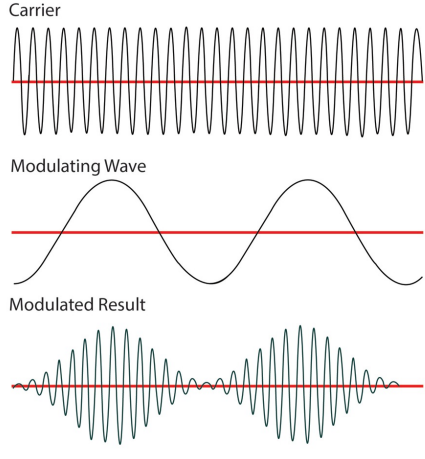

Figure 2: The modulated result is the summation of the carrier-wave and the modulating wave.

universal and collective whereas the contents of psyche are specific and individual. The universality of the proposed fabric of psyche makes it suitable for mathematical modeling.

4. Some Basic Properties of Hilbert Space

Hilbert spaces are special cases of vector spaces. Fig. 3 shows a very simple three-dimensional vector space. In vector-space mathematics, a distinction is made between vectors and scalars. Vectors can be thought of as arrows having both a length and a direction, whereas scalars are just numbers and only have a value. To distinguish between them, the notation of a vector is composed of a letter between a vertical bar and an angle bracket.[6] A represents a scalar and $|A\rangle$ represents a vector.

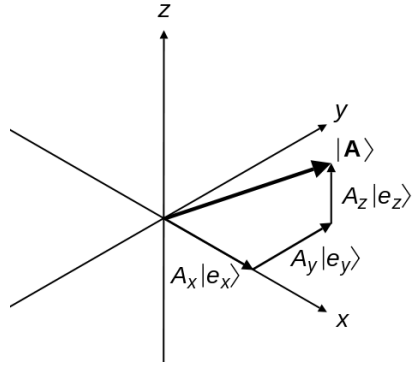

Figure 3: Example of a vector $|A\rangle$ in a three-dimensional vector space.

Figure 3 shows a three-dimensional vector space XYZ with three axes X, Y, and Z. The three vectors $|e_x\rangle$, $|e_y\rangle$ and $|e_z\rangle$ have the same length but different directions. Their directions are parallel to the X , Y and Z axes, respectively. The scalars A_x, A_y, and A_z can be multiplied with $|e_x\rangle$, $|e_y\rangle$ and $|e_z\rangle$, respectively, resulting in the vectors $A_x|e_x\rangle$, $A_y|e_y\rangle$ and $A_z|e_z\rangle$ which are aligned with the X, Y, and Z axes.

An important property of vector spaces is that the addition of vectors multiplied with scalars leads to a vector which also belongs to the same vector space. Adding vectors corresponds to aligning vectors head-to-tail. It can easily be seen from Fig. 3 that:

$$|A\rangle = A_x|e_x\rangle + A_y|e_y\rangle + A_z|e_z\rangle \tag{1}$$

[6]Different notations are possible. We use Dirac's notation which is common in quantum physics.

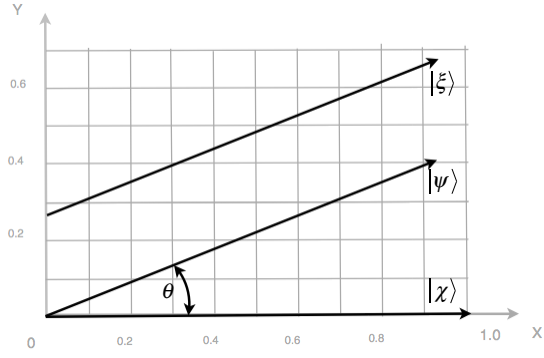

Figure 4: Inner product of vectors in a two-dimensional vector space.

Vectors can also be written as columns of numbers. In our example this would look as follows:

$$|e_x\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad |e_y\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad |e_z\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \tag{2}$$

4.1 Inner Product

A Hilbert space is a vector space in which the so-called inner product is defined.[7] The inner product can be interpreted as a degree of "parallelism" between vectors and always has a value between 0 and 1. The more the vectors point in the same direction the closer their inner product is to 1. Parallel vectors have inner product 1 whereas vectors which are perpendicular to each other have inner product 0.

In the example illustrated in Fig. 4 the inner product of vectors $|\chi\rangle$ and $|\psi\rangle$ is denoted as $\langle\chi|\psi\rangle$. In this simple two-dimensional case the inner product is proportional to the cosine of the angle θ included by vectors $|\chi\rangle$ and $|\psi\rangle$:

$$\langle\chi|\psi\rangle = \cos\theta \tag{3}$$

$|\xi\rangle$ and $|\psi\rangle$ in Fig. 4 are parallel vectors. Their respective inner products with any third vector, for example $|\chi\rangle$, are therefore the same no matter how distant $|\xi\rangle$ and $|\psi\rangle$ are from each other:

$$|\psi\rangle \neq |\xi\rangle \quad \text{yet} \quad \langle\chi|\psi\rangle = \langle\chi|\xi\rangle \tag{4}$$

[7] A detailed definition of Hilbert spaces is beyond the scope of this paper. For an authoritative monograph see Halmos (1951).

So far our illustrations of the defining properties of Hilbert spaces have been two- and three-dimensional. They can easily be generalized for vectors with an arbitrarily high dimension. For example, a state vector $|\psi\rangle$ in an n-dimensional Hilbert space can be written as a matrix of n columns and m rows:

$$|\psi\rangle = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix} \tag{5}$$

In this way, complex multidimensional vectors and Hilbert spaces can be constructed which can be combined in exactly the same way illustrated above.

4.2 Operators

In Hilbert-space mathematics, operators O are defined which transform vectors and spaces into different ones. In the formula

$$|\psi\rangle = O|\phi\rangle \tag{6}$$

the operator O "acts on" $|\phi\rangle$, thereby transforming $|\phi\rangle$ into $|\psi\rangle$. An operator O can be interpreted as a protocol to transform length and direction of vectors such as vector $|A\rangle$ in Fig. 3. The particular way in which the vectors are transformed is specified by the composition of O. Operators can mathematically be treated exactly the same as vectors.

4.3 Eigenvectors and Eigenvalues

The notions of eigenvectors and eigenvalues were coined by Hilbert. Their prefixes originate from the German "eigen", meaning "own". A vector $|\psi\rangle$ is an eigenvector of O if

$$O|\psi\rangle = \lambda|\psi\rangle \tag{7}$$

Such an equation is called an eigenvalue equation and λ is called an eigenvalue of O. Eigenvectors $|\psi_i\rangle$ of an operator O are vectors whose direction is not changed by O. In other words: $|\psi\rangle$ is not rotated by O. The specific eigenvectors and corresponding eigenvalues of an operator can be calculated by somewhat complex but straightforward mathematics.[8] Operators generally have multiple eigenvectors $|\psi_i\rangle$ and corresponding eigenvalues $\lambda_i (i = 1, 2, ...)$.

In quantum theory, when measuring a system in state $|\psi\rangle$, the probability w_k that eigenvector $|\phi_k\rangle$ with corresponding eigenvalue λ_k will be

[8] *Not all* operators have eigenvectors and eigenvalues, only operators meeting certain criteria do. Again, a detailed discussion of this is beyond the scope of this paper.

found as a result of the measurement is proportional to the square of the inner product of $|\psi\rangle$ and $|\phi_k\rangle$:

$$w_k = \langle\phi_k|\psi\rangle^2 \tag{8}$$

4.4 Fragmentation, Projection, Fusion

Three other properties of Hilbert spaces which are important for our purpose are:

- Fragments of a Hilbert space constitute again a (smaller) Hilbert space, called a subspace.
- Hilbert-space mathematics provides a simple projection protocol for projecting larger Hilbert spaces onto smaller subspaces.
- Two Hilbert spaces H_j and H_k can be combined in such a way that the product is a new larger Hilbert space $H_{jk} = H_j \otimes H_k$. The operation symbolized by $\otimes$ is called the outer product.

5. Correspondences between Hilbert Spaces and Psyche

It is proposed to formulate the phenomena of psyche and its mental states in mathematical terms as follows.

Each element i of the psyche of an individual j, be it conscious, unconscious, static or dynamic, be it a thought, a dream, a reflection, a states of reverie, a feeling, a memory or any state which is a combination of these, is describable by a state vector $|\psi_{ij}\rangle$ such that $|\psi_{ij}\rangle$ is an element of a Hilbert space H_j which corresponds to the totality of the psyche of this individual. The process of transformation of any mental state $|\phi\rangle$ into another mental state $|\psi\rangle$ is representable by an operator O such that $|\psi\rangle = O|\phi\rangle$.

This phenomenological formulation is largely consistent with the pertaining literature in recent Hilbert-space approaches in cognitive systems theory (see references quoted in the introduction above). From the psychoanalytical perspective formulation the following remarks are additionally useful.

I. The proposed formulation is consistent with the experience that things can "add up" in the psyche. For example, when $|e_1\rangle$, $e_2\rangle$, and $|e_3\rangle$ in Fig. 3 correspond to basic mental experiences such as "to the right", "backwards" and "up", then $|A\rangle$ corresponds to an experience which is a mixture of these basic experiences.

II. Any mixed mental experience can be decomposed into more basic experiences. This corresponds mathematically to writing $|A\rangle$ in terms of the basic experiences $|e_1\rangle$, $e_2\rangle$, and $|e_3\rangle$ as in equation (1).

III. In Hilbert space the coordinate system in which a state vector is defined can be freely rotated. This corresponds to the ability of the mind to reflect on a specific mental experience from different perspectives without changing the object itself. This is the essence of "eidetic reduction" in the phenomenology of Husserl; see Husserl (1913).

IV. Any Hilbert space can be projected onto a subspace. Psychological projection could thus be understood as a partial fusion of the psyches of two individuals and the mathematical projection onto a subspace of the joint Hilbert space.

V. Complex mental images require complex state vectors for their representation. Therefore it is important that any mathematical model of psyche is flexible enough to deal with this kind of complexity. The rules of Hilbert-space mathematics apply to arbitrarily complex vectors. Therefore, the most complex images, motifs, memories, sensory experiences and their combinations can be represented by vectors and operators in Hilbert space.

VI. In Hilbert-space mathematics, a subset of a Hilbert space is also a Hilbert space. This is consistent with the clinical experience of psychological "complex formation." When a person is "in a complex ", it seems to him *as if* he is experiencing his whole self and not a fragment. The latter could be understood by the fact that any fragment of a Hilbert space has the same combinatorial properties as the original whole Hilbert space, and therefore resembles the whole.

VII. The clinical experiences of transference, counter-transference, collusion, group-think and *folie-a-deux* can all be described in terms of Hilbert-space mathematics. In the mind of the patient or analyst who is conscious of this kind of dynamics these phenomena would be classical separable phenomena corresponding to the outer product property of Hilbert spaces as expressed in Sec. 4.4. In the mind of the patient or analyst who is unconscious about this dynamics, however, this would correspond to an irreducible entanglement, also describable in terms of the Hilbert-space model.

VIII. Unmetabolized material of the psyche (Bion's β-elements) corresponds to superposition states. In quantum physics, different systems of which the wavefunctions are superpositions can to a degree be entangled no matter how far they are removed from each other. In this sense, synchronistic experiences would thus no longer be an unexplainable meaningful correspondence but rather appears as a natural consequence of the properties of psyche.

A particularly interesting feature of Hilbert-space models is the formalism of eigenvalues and eigenvectors as expressed in Sec. 4.3. This

formalism appears to be a mathematical correlate to the mental phenomena of transforming ungrasped ambiguous mental material into grasped and unambiguous ideas and understanding.

The psychological process of finding expression for a not yet fully metabolized mental state $|\psi\rangle$ can mathematically be described by finding solutions of the eigenvalue equation of an operator O. The result of a conscious examination of an ungrasped mental content $|\psi\rangle$ is by necessity always one of the eigenvectors $|\psi_i\rangle$ of O.

- Every operator O can be associated with a question asked during a process of reflection. In its composition every particular operator O somehow already yields the possible "answers" of the question asked.[9] The result of the "asking of a question" is always expressed in terms of previous experiences.
- The mathematics of the grasping process is similar to the collapse of the wavefunction in quantum theory. A superposition state is reduced to an unambiguous state. Thus every metabolized thought or idea can be seen as a reduction of the primary unmetabolized material. In general, O has multiple eigenvectors but only one of them will "appear" as the result of the "examination". The question which eigenvalue will appear could be answered statistically, analogous to quantum physics (see equation 8): the probability w_k that eigenvector $|\phi_k\rangle$ with corresponding eigenvalue λ_k will be found as a result of the reflective examination is proportional to the square of the inner product of $|\psi\rangle$ and $|\phi_k\rangle$: $w_k = \langle\phi_k|\psi\rangle$.
- The implication of this postulate would be that conscious reflective investigation of unmetabolized mental content would not be a neutral operation but would change the "system under observation". This corresponds clearly with the clinical experience that focussed introspection has transformative power; see Jung (1970).
- The operator O can be seen as merely describing mental change, but it may also be seen as prescribing change. In the latter case O could be the mathematical term describing a psychological agency, a "force", a motif, or an archetype.

6. Discussion

The phenomena of *mental grasping, metabolization* and *conceptualization* are central in understanding human mental functioning and are the

[9] This corresponds to Heidegger's idea that asking a question already requires possessing part of the answer (Heidegger 1962, pp. 21ff). In quantum mechanics, von Neumann (1955) emphasized the importance of asking the question as a first step in the measurement process. Stapp (2007) locates this first step of von Neumann in the mind-brain.

subject of model-formation in a variety of disciplines such as neuroscience, cognitive psychology and psychoanalytic psychology. Particular progress has been made in the last decennia with the development of frameworks for conceptualization (e.g., Margolis and Laurence 1999) and a body of research on algebraic structures fits these frameworks (e.g. Widdows and Peters 2003). It is plausible to believe that the formalization of metabolization as proposed here is compatible with the recent body of work on quantum cognition. Both are based on a type of ortho-algebra similar to (if not identical with) the Hilbert-space algebra used in quantum theory.

The proposed idea also correlates with neuroscientific findings concerning the different experiences mediated by the two brain hemispheres. McGilchrist (2009, p. 174) concludes:

> The world of the left hemisphere is dependent on denotative language and abstraction, yields clarity and power to manipulate things that are known fixed, static, isolated, decontextualized, explicit, disembodied ... [while the right hemisphere] yields a world of individual, changing, evolving, interconnected, implicit .... never fully graspable, always imperfectly known.

There are remarkable correspondences between the experience mediated by the right hemisphere and the general properties of a superposition state, as well as between the experience mediated by the left hemisphere and the properties of a collapsed state.

Furthermore, the world of the right brain grounds that of the left brain world and is primary to it.[10] Together, the correspondences between right-hemisphere and superposition states, left-hemisphere and single states, and the finding that the right hemisphere is primary to the left hemisphere suggest that the process of conceptualization and metabolization involves a complex dynamics between the hemispheres. It might be mathematically represented by an operation described by an eigenvalue equation, such that the unmetabolized states correspond to superposition states and the metabolized states correspond to the resulting eigenvectors.

Access to metabolized states with grasped content is much easier than access to unmetabolized blurred states. It is intuitive to associate conscious states with grasped content with eigenstates – but how intuitive is it to associate unconscious states with superpositions? Answering this question is hampered by the inaccessibility of the unconscious. However, in analytical psychology the unconscious is believed to be somewhat observable in dreams and, in psychologically projected form, in mythology and alchemy. We could therefore ask if there are "superposition states" in dreams, mythology or alchemy.

[10] McNeill (2000, p. 326) summarizes: "There is a global-synthetic image ... [when] ... there is not yet a linguistic structure ...".

For example, a dreamed house can be the parental house and current house of the dreamer at the same time. The same is true for persons that appear in dreams. For example, it is very common that patients speak about a dream in which a man occurs who is at the same time the dreamer's father and neighbor.

In mythology and alchemy, "superposition states" are abundantly present as well. For example, in Egyptian mythology Isis is the wife, mother and daughter of Osiris. Alchemical texts which were not yet "spoiled" by the rational thinking of the enlightenment can be particularly useful to investigate the nature of the unconscious.[11] For example, pairs of strict opposites such as king and queen appear in alchemical texts as transcending unities such as the image of the Rebis or Hermaphrodite and are referred to as *coincidentia oppositorum*. A Jungian psychological interpretation of these texts is that the "unities holding opposites" are central in the unconscious and precede transcendence (see Jung 1985, par. 522). Through an alchemical lens the superposition state is a most perfect coincidence of opposites.

Although these parallels from ancient and modern disciplines are striking and intriguing, our proposal that the process of mental metabolization can be modeled with eigenvalue equations does not rise above the status of a promising hypothesis at best. It would be of great value, therefore, if specific experiments could be designed for testing the idea. It seems that such empirical work would require inducing defined unmetabolized states in test subjects. Possibly such tests could involve the subliminal exposure of individuals to audio or video signals in superposition, i.e. signals that are well-defined superpositions of signals that could be considered as "eigen-signals". In fact, if such an experiment could be designed, we would expect the result to be not a superposition but a single state. More specifically, for a large population of test subjects we would expect a statistical distribution of reported single states which would quantitatively correspond to the superposition due to the composed stimulus.

If the metabolization process were indeed be based on the solution of eigenvalue equations, this would entail implications for psychoanalytical practice as well as for the interpretation of quantum physics. Some interesting points for analytical practice are:

- Finding words or images for expressing unmetabolized material is crucial. Although the clinical importance of this has been known for a long time, the present model helps to appreciate this even better. It is impossible to solve an eigenvalue equation without finding the eigenvectors. The parallel in psychoanalysis is that it is impossi-

[11] According to Jung the alchemists were not primarily seeking ways of making material gold but rather had "spiritual gold" as their foremost aim: *Aurum Nostre non est aurum Vulgi*.

ble to metabolize the unconscious without finding the appropriate words, images or motifs.

- Splitting, complex formation, collusion, projection and even synchronicity can be understood as basic, mathematically expressible properties of psyche. This insight might help patients to accept these phenomena as "normal".
- To help patients tolerating ambiguity and states of not knowing are an important ingredient of psychoanalytical practice. In the proposed model unmetabolized material can be understood as a direct manifestation of mental states in superposition. Perhaps this insight might help patients to accept their own states of "ignorance".

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